Eigenmode Analysis of the Thermoacoustic Combustion Instabilities Using a Hybrid Technique Based on the Finite Element Method and the Transfer Matrix Method

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Abstract

Thermoacoustic combustion instabilities affect modern gas turbines equipped with lean premixed dry low emission combustion systems. In the case of annular combustion chambers, experimental test cases carried out on small scale test rigs equipped with single burner arrangements fail to give adequate indications for the design of the full scale combustion chamber, since they are unable to reproduce the interaction of the flame fluctuation with the azimuthal pressure waves. Therefore there is a large interest in developing techniques able to make use of data gathered from tests carried out on a single burner for predicting the thermoacoustic behavior of the combustion chamber at full scale with its actual geometry.

A hybrid technique based on the use of the finite elements method and the transfer matrix method is used to identify the frequencies at which thermoacoustic instabilities are expected and the growth rate of the pressure oscillations at the onset of instability, under the hypothesis of linear behavior of the acoustic waves. This approach is able to model complex geometries such as annular combustion chambers equipped with several burners. Heat release fluctuations are modeled through a classical n-τ Flame Transfer Function (FTF). In order to model the acoustic behavior of the burners, the computational domain corresponding to each burner is substituted by a mathematical function, that is the Burner Transfer Matrix (BTM), that relates, one to each other, pressure and velocity oscillations at either sides of the burner. Both the FTF and the BTM can be obtained from experimental tests or from CFD simulations. The use of the transfer matrix permits us to take into account parameters, such as the flow velocity and the viscous losses, which are not directly included in the model.

This paper describes the introduction of the burner transfer matrix in the combustion chamber model. Different geometries of combustion chamber and burner are tested.

The influence of the parameters characterizing the transfer matrix is investigated. Finally the application of the BTM to an actual annular combustion chamber is shown.

Keywords

Hybrid Technique; FEM; Transfer Matrix Method; Thermoacoustics; Burners; Eigenvalues

Nomenclature

Latin:

A	cross sectional area
b	length
c	speed of sound
F	Riemann invariant
G	Riemann invariant
i	imaginary unit
k	acoustical wave number
l	length
LHV	lower heating value
M	Mach number
n	interaction index
p	pressure
q	volumetric heat release rate
Q	rate of heat release per unit area
R	gas constant
RR	rate of reaction
Re	real part
s	thickness

t	time
T	temperature
u	velocity vector
x	position vector

Greek:

α	area ratio
β	parameter
γ	ratio of specific heats
δ	Dirac's delta
ζ	pressure loss coefficient
λ	eigenvalue $=-i\omega$
ϱ	density
σ	time lag distribution
τ	time delay
ω	angular frequency

Subscripts:

d	downstream
eff	effective
i	reference position
u	upstream

Introduction

The introduction of lean premixed burners in gas turbine systems for power generation has determined an increase of the risk of thermoacoustic instabilities [1,2,3]. This phenomenon is caused by self-sustained pressure oscillations, which arise from coupling of heat release fluctuations and acoustic waves propagating within the combustion system. These instabilities are dangerous because can lead to strong vibrations of the system with consequent structural damages. The phenomenon is known since a long time and several models and techniques have been proposed over the years to have a better comprehension of its causes and to predict which operating conditions are more prone to cause it.

Quite often low-order models [1,4,5,6,7] are proposed to represent the combustion instability in the combustion system modeled as a network of acoustic elements (and so the as much used name of Acoustic Networks), where each element corresponds to a component, such as a duct, a nozzle, a burner, etc...

Generally, each component represents a one-dimensional propagation of the acoustic waves, mainly within the limits of the linear acoustic and harmonic time dependence ($i\omega t$). In case of the application to annular chambers, two-dimensional components are also proposed to model both axial and azimuthal waves [8,9]. These tools may be used to study the pressure oscillations by dividing the combustor into several zones and representing the acoustic field in each zone. Hence, a combustor system is defined as a series of subsystems, using mathematical transfer function matrices to connect these lumped acoustic elements one to each other, so providing the continuity of acoustic velocity and pressure across each zone. The unknowns are acoustic pressure and velocity fluctuations (p' and u' respectively) at the ports of each network element. In this scheme flame is concentrated in one of the lumped elements and located at the beginning of the combustion chamber. Flame transfer function (FTF) is written as the ratio of heat release rate oscillation to inlet velocity fluctuation, where both of them are normalized by their corresponding time-averaged values. These models are very helpful to understand the instability mechanisms, but when complex three-dimensional geometries are involved, these models become unable to model the propagation of the acoustic waves adequately.

LES (Large Eddy Simulation) codes are proposed by several authors [10,11,12,13,14] to investigate the phenomenon of combustion instability and matching pressure oscillations with turbulent combustion phenomena, considering the dynamically relevant scales during the resolution, related to fluid dynamic phenomena and the kinetic chemistry. The disadvantages are concerned with the enormous computational efforts required for the resolution.

A finite element method (FEM) approach may also be used to solve the acoustic problem in three-dimensional geometries [15,16,17]. Although some authors proposed the acoustic analysis in the time domain in order to model the onset of the perturbations and how they increase reaching the limit cycle [15], in [16,17] the complex eigenfrequencies of the system are detected to ascertain if the corresponding modes are stable or unstable [17]. This approach numerically solves the differential equation problem converted in a complex eigenvalue problem in the frequency domain. The eigenvalue problem is nonlinear and it is solved by means of a linearization under the hypothesis of small oscillations.

In this paper a hybrid technique is described, since the combination of lumped elements and three-dimensional elements is achieved. The numerical procedure, able to detect the complex eigenvalues of the analyzed system in presence of a flame transfer function and a transfer matrix function, is implemented within commercial software, COMSOL Multiphysics, based on the finite element method. The main idea is to introduce in the FEM framework the transfer matrices of the low-order models to model the burners. Each burner is described not as a computational domain, but as a mathematical function, which can be obtained analytically or experimentally. Additionally, this idea permits us to take into account the influence of parameters, such as the mean flow and the viscous losses, which are not considered in the FEM framework. A strong reduction in the computational time is also obtained.

The goal of this study is to provide adequate indications in the design stage of a full scale combustion chamber. In fact, there is a great interest on developing methods able to make use of the data gathered from experimental or numerical tests carried out on a single burner for predicting the thermo acoustic behavior of a practical combustion chamber.

In the first section the mathematical model is introduced. In the second section the application of the transfer matrices is described and the applications to different geometries of the combustion chamber are shown: cylindrical and annular configurations. The influence of parameters characterizing the flame model and the transfer matrix is also investigated. In the third section the transfer matrix method is applied to a practical annular combustion chamber.

Mathematical Model

As regards gas turbine combustion systems, the flow velocity is generally far below the sound velocity, both in the plenum placed at the exit of the compressor and in the combustion chamber, while the flow velocity is not negligible in some limited areas, such as conduits of single burner or multiple burners that connect the plenum to the chamber. These areas, in terms of propagation of pressure waves, can be treated as "compact" elements that can be modeled by means of specific transfer function matrices, obtained experimentally or numerically through CFD or aeroacoustics codes [18,19,20,21]. The hybrid method proposed here makes a combined use of

- FEM for the analysis of the propagation of the acoustic wave in three dimensional domains with negligible flow velocity,

- Transfer matrices to model the acoustic waves in the ducts of the burners, where there is a one-dimensional propagation of the waves, in presence of not negligible levels of flow velocity.

In this section the mathematical model adopted for the FEM analysis of the volumes with negligible flow velocity, even in presence of heat release fluctuations and temperature gradients, is described.

In the acoustic analysis each variable is assumed to be composed of a steady mean value (identified by an overbar) and a small perturbation (identified by a prime): $p(\mathbf{x},t) = \bar{p}(\mathbf{x}) + p'(\mathbf{x},t)$ and $\mathbf{u}(\mathbf{x},t) = \bar{\mathbf{u}}(\mathbf{x}) + \mathbf{u}'(\mathbf{x},t)$. The flow velocity is considered negligible in comparison with the sound velocity, within the computational domain, so $\bar{\mathbf{u}}(\mathbf{x}) = 0$ in the whole domain. The effects of viscous losses and thermal diffusivity can also be neglected, while the temperature can vary within the domain under the simplifying hypothesis that the fluid can be considered as an ideal gas, which means that specific heats are supposed constant and the variation of the composition caused by combustion does not produce a variation of the number of moles. From these hypotheses, in presence of heat fluctuations, the linearized equations for the perturbations can be obtained [8,22], giving the inhomogeneous wave equation

$$\frac{1}{\bar{c}^2}\frac{\partial^2 p'}{\partial t^2} - \bar{\rho}\nabla \cdot \left(\frac{1}{\bar{\rho}}\nabla p'\right) = \frac{\gamma - 1}{\bar{c}^2}\frac{\partial q'}{\partial t}. \qquad (1)$$

where c is the speed of sound and q' is the fluctuation of the heat input per unit volume. The term at the RHS of Eq.(1) shows that the rate of non-stationary heat release creates a monopole source of acoustic pressure disturbance. Considering that mean flow velocity is neglected, no entropy wave is generated and the pressure fluctuations are related to the velocity fluctuations by

$$\frac{\partial \mathbf{u}'}{\partial t} + \frac{1}{\bar{\rho}}\nabla p' = 0. \qquad (2)$$

This work is conducted in the frequency domain, so that each variable fluctuation is expressed as a complex function of time:

$$p' = \mathrm{Re}(\hat{p}(\mathbf{x})\exp(i\omega t)) \qquad (3)$$

where the hat refers to a complex variable and the same is for the other variables. Introducing this condition into Eq.(1), the Helmholtz equation can be obtained:

$$\frac{\lambda^2}{\overline{c}^2}\hat{p} - \overline{\rho}\nabla\cdot\left(\frac{1}{\overline{\rho}}\nabla\hat{p}\right) = -\frac{\gamma-1}{\overline{c}^2}\lambda\hat{q} \qquad (4)$$

where $\lambda = -i\omega$ is the eigenvalue, being ω a complex variable, comprising a real part that gives the frequency of oscillations and an imaginary part that gives the growth rate at which the amplitude of oscillations increases per cycle. Eq.(4) shows a quadratic eigenvalue problem which can be solved by means of an iterative linearization procedure. COMSOL Multiphysics uses the Arpack Fortran as numerical routines for large-scale eigenvalue problems. It is based on a variant of the Arnoldi algorithm, called the implicit restarted Arnoldi method [23]. An iterative procedure based on a quadratic approximation around an eigenvalue linearization point λ_0 is adopted. Such a procedure is speeded up by using, as approximate starting eigenvalue, the value obtained through the analysis of the system without heat release fluctuations. The solver reformulates the quadratic eigenvalue problem as a linear eigenvalue problem of the conventional form $Ax = \lambda Bx$, and, iteratively, updates the linearization point until the convergence is reached. The software approximates the nonlinear model with a linear model, and the approximation is valid only when the solution is close to the linearization point. This means that not always the obtained solution is the exact solution, but it could be a spurious solution, so that a manual re-initialization is needed.

Transfer Matrix Method

The transfer matrix is a 2x2 matrix whose coefficients can relate the fluctuations of acoustic pressure p' and velocity u' at one junction, or port of the element, to the fluctuations of acoustic pressure and velocity on the other junction of the element. Instead of the physical acoustic variables, the Riemann invariants F and G are usually adopted. Riemann invariants are related to acoustic pressure and velocity by

$$F = \frac{1}{2}\left(\frac{p'}{\overline{\rho}c} + u'\right), \qquad G = \frac{1}{2}\left(\frac{p'}{\overline{\rho}c} - u'\right) \qquad (5)$$

and can be thought as waves propagating, respectively, in the downstream and upstream direction. For a simple duct of uniform cross section, under the

hypotheses of plane waves without mean flow and dissipative effects, the transfer matrix is given by [24]:

$$\begin{pmatrix} F_d \\ G_d \end{pmatrix} = \begin{pmatrix} \exp(ikl) & 0 \\ 0 & \exp(-ikl) \end{pmatrix}\begin{pmatrix} F_u \\ G_u \end{pmatrix} \qquad (6)$$

where $k = \omega/c$ is the wave number and l is the length of the duct.

Test Cases

At the beginning, some preliminary application tests are carried out on a duct with uniform cross-section area, uniform temperature and no mean flow. This scheme is characterized by an open-end inlet ($p'=0$) and a closed-end outlet ($u'=0$), see Fig. 1. A planar flame sheet is supposed to be located at the abscissa $x=b$. Such a scheme is the same proposed by Dowling and Stow [8]. In Fig. 2, the scheme A represents the computational domain of conventional "one-piece" duct, while the computational domain of the scheme B is divided in two pieces connected by means of the transfer matrix, given in Eq.(6), where l represents the length of the piece of the duct substituted by the transfer matrix. In this one-dimensional case, the planar wave hypothesis can be adopted and only the abscissa x along the duct is used to describe the variation of acoustic pressure and velocity in the duct.
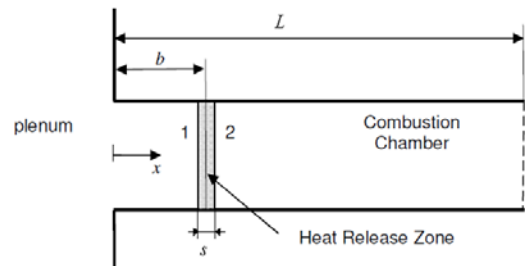

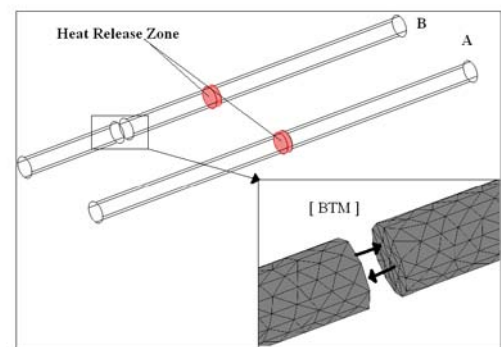FIG. 1 SCHEME OF A STRAIGHT DUCT WITH UNIFORM CROSS SECTION

FIG. 1 COMPUTATIONAL GRIDS OF THE STRAIGHT DUCTS WITH UNIFORM CROSS SECTION. DUCT A IS WITHOUT BTM, DUCT B IS WITH BTM

Test Case 1

In the first test case the transfer matrix method is applied to the described scheme without flame fluctuations. The results obtained by means of the application of the proposed method are compared with those obtained from the same scheme without the interruption under the same operating conditions.
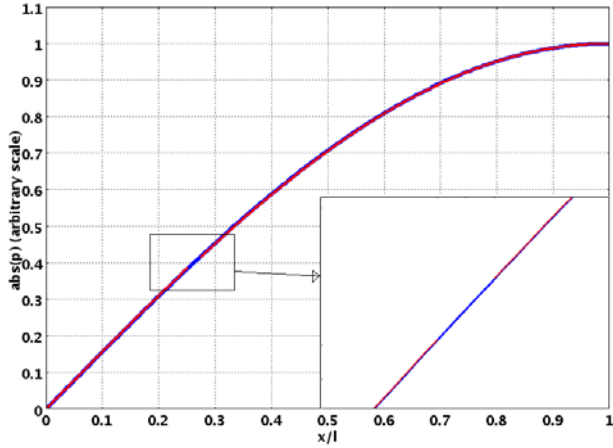

FIG. 2 MODESHAPE CORRESPONDING TO THE FIRST EIGENMODE OF THE STRAIGHT DUCT WITH UNIFORM CROSS SECTION (FIG. 2) WITH UNIFORM TEMPERATURE. RED LINE FOR DUCT B, BLUE LINE FOR DUCT A

For comparison, in Fig. 3, the mode shapes corresponding to the first eigenfrequency are shown for both the schemes, A and B. It appears that there is a very good agreement between the results obtained with transfer matrix and those obtained from the original "one-piece" duct. The correspondence of the data is highlighted in the zoom of acoustic pressure path. The first eigenfrequency of this system results equal to 21.4 Hz in both cases. Pressure and velocity fluctuations on one side are well linked to the corresponding on the other side of the interruption. The results confirm that, in such a simple geometry, without changes of cross section geometry and area, where only plane waves are present and no tri-dimensional effects, the transfer matrix does not determine any alteration of the modeshapes.

Test Case 2

In the second test case the transfer matrix method is applied to the described scheme including flame fluctuations. Following the approach used in [8], the rate of heat release fluctuations per unit volume $q'(x,t)$ is assumed to be related to the local velocity u' upstream the flame zone with a time delay τ, by:

$$q'(x,t) = Q'(t)\delta(x-b) \qquad (7)$$

$$Q'(t) = -\frac{\beta\overline{\rho c}^2}{\gamma-1}u_1'(t-\tau) \qquad (8)$$

where $Q'(t)$ is the rate of heat input per unit area of the cross section of the duct and subscript 1 denotes conditions just upstream of this region of heat input: $u_1'(t) = u'(x_1^-,t)$, where $x_1 = s - b/2$ (see Fig. 1). Eq.(7) relates the fluctuations of heat input rate per unit volume, $q'(x,t)$, to the fluctuations of heat input rate per unit area of the cross section, $Q'(t)$, through the Dirac's delta $\delta(x-b)$. The non dimensional parameter β gives a measure of the coupling between heat release and velocity fluctuations. In the FEM eigenvalue analysis, heat release fluctuations are assumed to occur in a thin volume with thickness s and the Dirac's delta $\delta(x-b)$ can be roughly approximated by

$$\delta(x-b) \cong \begin{cases} 0 & x \leq b-s/2 \\ 1/s & b-s/2 < x \leq b+s/2 \\ 0 & x > b+s/2 \end{cases} \qquad (9)$$

Considering Eq.(3), after some algebraic steps, Eq.(4) becomes

$$\frac{1}{\overline{\rho c}^2}\lambda^2\hat{p} - \frac{1}{\rho}\nabla\cdot(\nabla\hat{p}) = \left[-\beta\delta(x-b)(-\lambda)\hat{u}(b^-)\exp(\lambda\tau)\right] \qquad (10)$$

that is the governing equation to be solved in the FEM code.

Fig. 4 shows the modeshapes for the first axial mode, considering that blue line is related to the whole "one piece" system, whereas red line is related to the system with transfer matrix.
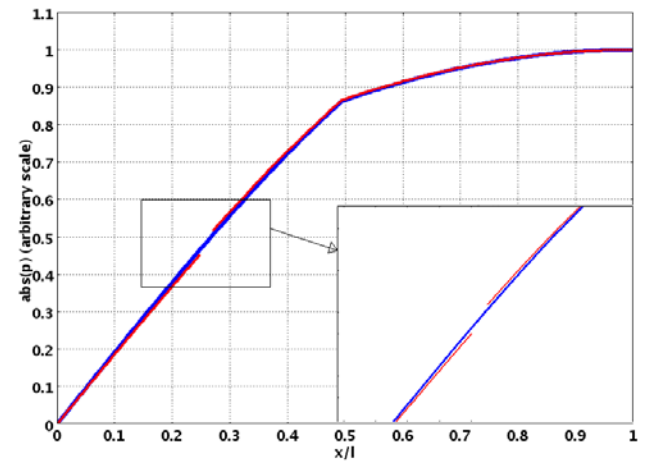

FIG. 3 MODESHAPES CORRESPONDING TO THE FIRST EIGENMODE OF THE DUCT IN FIG. 2 WITH T_2/T_1=2.14 AND B/L=0.3. RED LINE FOR DUCT B, BLUE LINE FOR DUCT A.

In Fig. 4 the position of the flame can be recognized by the presence of a knee in the modeshape. Modeshapes with and without transfer matrix are nearly coincident

and the differences can be considered negligible. Table 1 highlights the good agreement of the results with and without the transfer matrix.

TABLE 1 VALUES OF THE FIRST EIGENFREQUENCIES CORRESPONDING TO THE EIGENMODES IN FIG. 4.

	b/L = 0.3	b/L = 0.5
with transfer matrix	(34.3 + 1.3i) Hz	(31.6 + 1.1i) Hz
without transfer matrix	(34.8 + 1.3i) Hz	(32.0 + 1.1i) Hz

Test Case 3

A little more realistic quasi one-dimensional combustor is now presented. The analyzed scheme is characterized by three cylindrical ducts, representing plenum, burner and combustion chamber, respectively. In this case, the burner has a diameter lower than that of plenum and combustion chamber, which, for simplicity, has the same diameter (Fig. 5), with a ratio between the length of the burner, y, and the overall length, L, is equal to 0.1. The boundary conditions at the inlet and at the outlet are assumed as acoustically closed ends, $u'(0) = 0$ and $u'(L) = 0$.

Fig. 6 shows the criteria adopted to substitute the burner with a transfer matrix: scheme A represents the original geometry, while schemes B and C represent two alternative geometries where a transfer matrix is used to model the removed volume of the burner. The difference between the schemes B and C concerns the surfaces adopted as interfaces between the transfer matrix (1D model) and the domain discretized by FEM (3D model). Such surfaces are colored in red in Fig. 6. In the scheme C both the interface surfaces are circles with the same diameter of the burner, whereas in the scheme B the interfaces have the same diameter of plenum and combustion chamber.

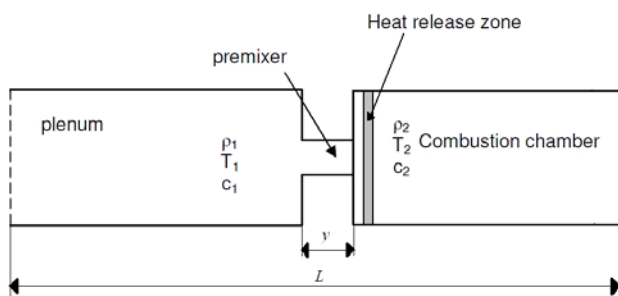Heat release fluctuations are modeled by Eq.(10).

FIG. 4 SCHEME OF A STRAIGHT DUCT WITH VARIATION OF SECTION.

Fig. 7 and Table 2 show that the application of

analytical transfer matrix to the system with variation of section yields very good results both in modeshapes and in identifying the complex eigenfrequencies.
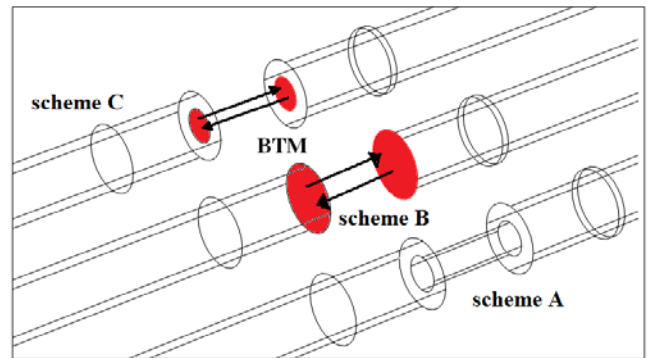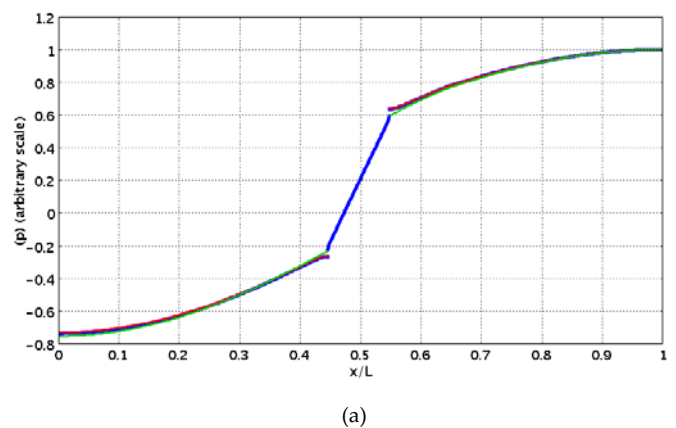

FIG. 5 DIFFERENT COMPUTATIONAL DOMAINS AND INTERFACES TO THE BTM (PARTICULAR). SCHEME A REPRESENTS THE ORIGINAL GEOMETRY, SCHEMES B AND C WITH BTM.

TABLE 2 VALUES OF THE EIGENFREQUENCIES.

Scheme	1st mode	2nd mode
A	(77.4 + 4.2i) Hz	(212.8 - 0.3i) Hz
B	(79.7 + 3.7i) Hz	(213.4 - 0.4i) Hz
C	(77.2 + 4.2i) Hz	(212.7 - 0.3i) Hz

Scheme C gives results better than B, because the effects of the radial propagation of the waves occurring in the original scheme A are well reproduced, as it is shown in Fig. 8, which clearly shows that scheme C is able to capture the radial component of the acoustic waves in proximity of the burner, whereas scheme B does not recognize such effects.

(a)

From this test case, it appears that if interfaces are correctly identified, the model with transfer matrix is able to reproduce very accurately the original system.
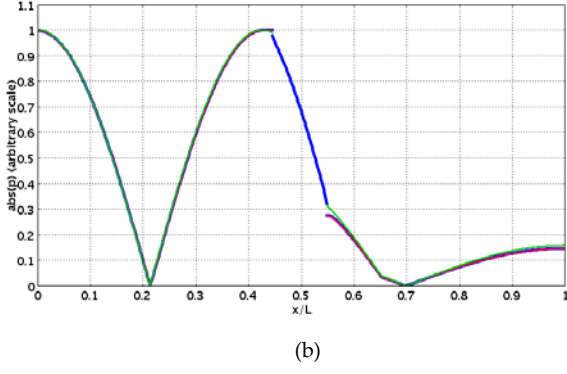
(b)

FIG. 6 MODESHAPES CORRESPONDING TO THE FIRST (A) AND SECOND (B) EIGENMODE OF THE DUCT WITH VARIATION OF SECTION (FIG. 6) AND $T_2/T_1=2$. RED LINE FOR SCHEME C, GREEN LINE FOR SCHEME B, BLUE LINE FOR SCHEME A
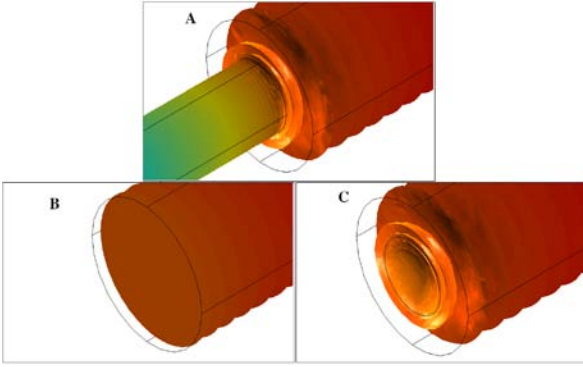

FIG. 7 ACOUSTIC PRESSURE FIELD FOR SCHEMES A, B AND C

Test Case 4

In the following test the burner is modeled by means of the transfer matrix proposed both by Fanaca [20] and Alemela [21] in order to model the burner used during their experiments at Technische Universitet of Munich. The transfer matrix is obtained assuming one-dimensional flow with low Mach number within a "compact element" represented by a duct having short length (compared to the wavelength), variable cross section and pressure losses. Linearizing the mass and momentum conservation equations, the following equations can be obtained

$$\left[A\left(\frac{\hat{p}}{\rho c}M + \hat{u} \right) \right]_u^d = 0 \qquad (11)$$

$$\frac{i\omega}{c}\hat{u}_u l_{eff} + \left[M\hat{u} + \frac{\hat{p}}{\rho c} \right]_u^d + \zeta M_d \hat{u}_d = 0 \qquad (12)$$

where M is the Mach number. In Eq.(12) the effective length l_{eff} given by

$$l_{eff} = \int_{x_u}^{x_d} \frac{A_u}{A(x)}\,dx, \qquad (13)$$

takes into account the inertia of the air mass in the duct, assuming that the Mach number is sufficiently low and that the effect of air compressibility can be neglected. The coefficient ζ gives the acoustical pressure losses and is generally close to the mean flow pressure loss coefficient, due to friction and flow separation $\zeta = 2\Delta p/(\overline{\rho u}^2)$. Using effective length l_{eff} and pressure loss coefficient ζ, neglecting higher order Mach number terms, the transfer matrix of a compact element is obtained from Eq.(11) and Eq.(12)

$$\begin{bmatrix} \dfrac{\hat{p}}{\rho c} \\ \hat{u} \end{bmatrix}_d = \begin{bmatrix} 1 & M_u - \alpha M_d(1+\zeta) - ikl_{eff} \\ \alpha M_u - M_d & \alpha + M_d ikl_{eff} \end{bmatrix} \begin{bmatrix} \dfrac{\hat{p}}{\rho c} \\ \hat{u} \end{bmatrix}_u \quad (14)$$

where $\alpha = A_u/A_d$ is the area ratio and $k = \omega/c$ is the wave number.

The transfer matrix of a burner modeled as a compact element by Eq.(14), is applied to the same one-dimensional combustion system examined in Test Case 3 (Fig. 5) with addition of pressure losses and not negligible Mach number. Fig. 9 shows the influence of the Mach number on the first eigenmode. In this way it is possible to carry on this procedure both in design stage and in check stage. In the design stage, one can simulate the process by varying the operating parameters in order to obtain the desired specimens. In the check stage, one can verify that the solutions adopted in order to avoid thermoacoustic combustion instabilities succeed in their aim, introducing the obtained experimental results or modeling these solutions inside the finite element code.

Additionally, the influence of some parameters (l_{eff} and ζ), characterizing the transfer matrix Eq.(14), on frequency and growth rate of the modes is conducted.
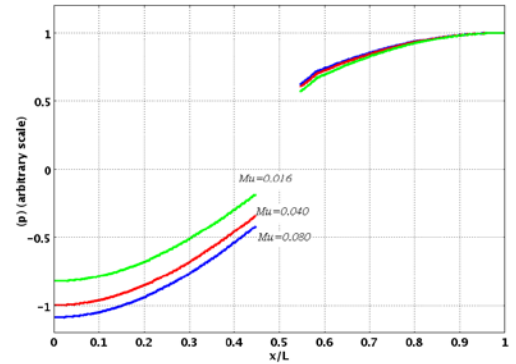

FIG. 8 MODESHAPES CORRESPONDING TO THE FIRST EIGENMODE OF THE STRAIGHT DUCT WITH VARIATION OF SECTION SHOWN IN FIG. 5, AND $T_2/T_1=2$, $Z=1.2$.
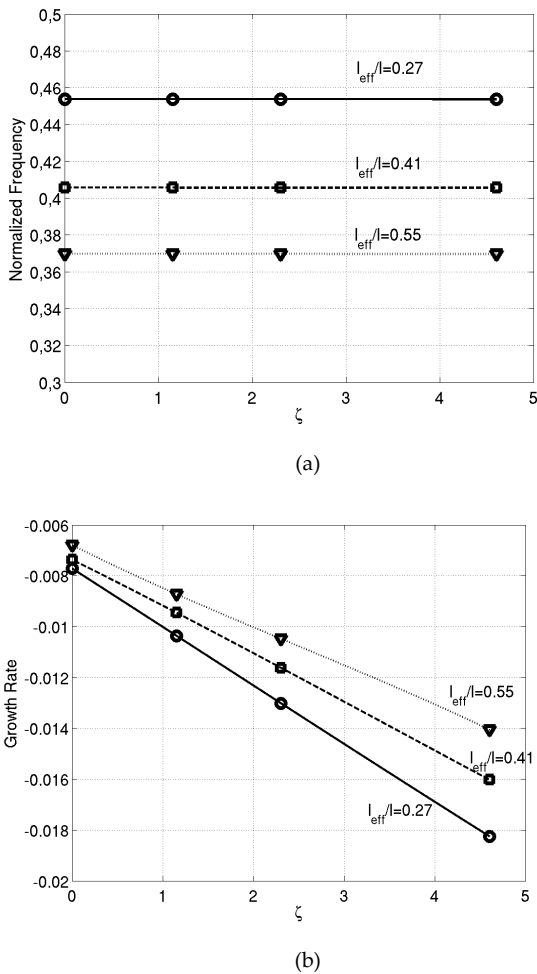
(a)

(b)

FIG. 9 INFLUENCE OF LOSS COEFFICIENT AND EFFECTIVE
LENGTH ON NORMALIZED FREQUENCY (A) AND GROWTH
RATE (B) OF THE FIRST EIGENMODE

Fig. 10a shows that frequency is influenced only by the effective length of the burner. In fact when the length of the burner increases, the frequency of the system decreases, but it is not influenced by the variation of the loss coefficient ζ. On the other hand, Fig. 10b shows that growth rate is influenced both by the loss coefficient ζ and by the effective length. The mode is stable for this configuration, but it becomes more stable when ζ increases, determining a larger damping. These results show that it is possible to avoid combustion instabilities correctly designing the shape of the burners taking into account the geometrical constraints of the combustion chamber. In this way a correct setting of the fluid dynamic conditions inside the burners have to be well considered, in order to avoid important decrement of static pressure.

Test Case 5

In this test case the configuration is the same of the

previous test, except the shape of the burner, which is conical (Fig. 11), with the ratio of cross-section area of the entrance to the burner and of the plenum equal to *0.562*, the ratio of cross-section area of the exit of the burner and of the combustion chamber equal to *0.125*. The burner is substituted by the transfer matrix of Eq.(14) and heat release fluctuations are considered. Two different conditions are analyzed to observe the influence of the Mach number and the influence of the pressure loss coefficient, ζ.
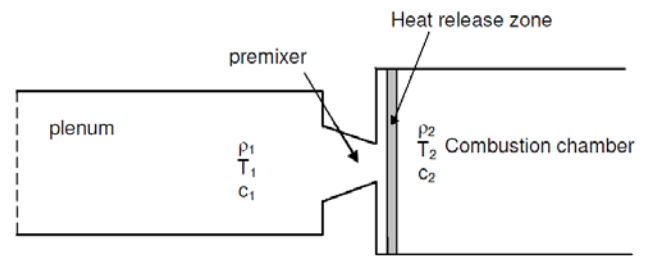

FIG. 10 SCHEME OF A STRAIGHT DUCT WITH VARIATION OF
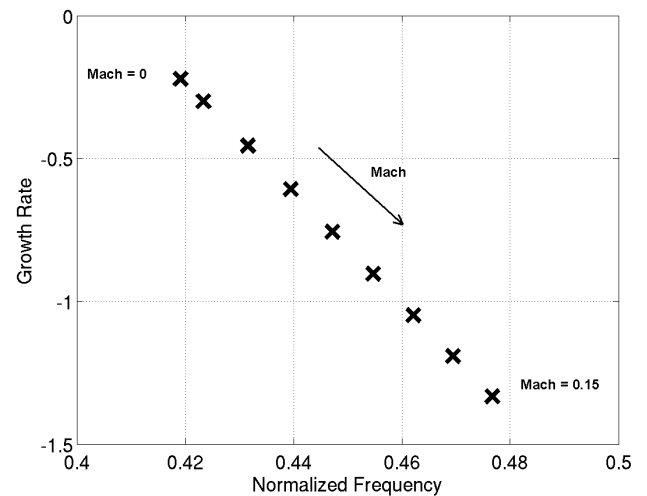SECTION AND CONICAL BURNER.

FIG. 11 VARIATION WITH MACH NUMBER OF FREQUENCY
AND GROWTH RATE OF THE FIRST EIGENMODE FOR THE
STRAIGHT DUCT WITH CONICAL BURNER (FIG. 11).

Fig. 12 shows the influence of the Mach number on frequency and growth rate with a fixed value of ζ. As in the previous test, growth rate decreases when Mach number increases. On the other hand, frequency increases with Mach number. Fig. 13 shows the influence of the pressure loss coefficient ζ with a fixed value of the Mach number. The trend is similar to the one obtained varying the Mach number.
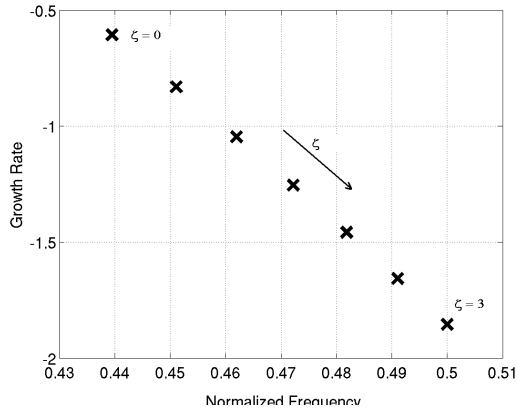
FIG. 12 VARIATION OF FREQUENCY AND GROWTH RATE OF THE FIRST EIGENMODE FOR $Z = 0$ TO 3 WITH AN INCREMENT OF 0.5 FOR THE STRAIGHT DUCT WITH CONICAL BURNER (FIG. 11).

Test Case 6

This test case concerns with a more complex geometry, that is the annular configuration examined by Pankiewitz and Sattelmayer [15]. The combustor is characterized by a diffusion chamber ring (plenum) and an annular combustion chamber connected by 12 cylindrical burners, Fig. 14. The mean diameter is $0.437\ m$, the external diameter of the plenum is $0.540\ m$ and of the combustion chamber is $0.480\ m$. The length of the plenum is $0.200\ m$ and of the combustion chamber is $0.300\ m$. Each burner has a diameter of $0.026\ m$ and a length of $0.030\ m$. Temperature in the combustion chamber is 2.89 times the temperature on the plenum. Flame is assumed to be concentrated in a narrow zone at the entrance of the combustion chamber, as shown in Fig. 14. Closed-end inlet and outlet are assumed as boundary conditions ($u'=0$).
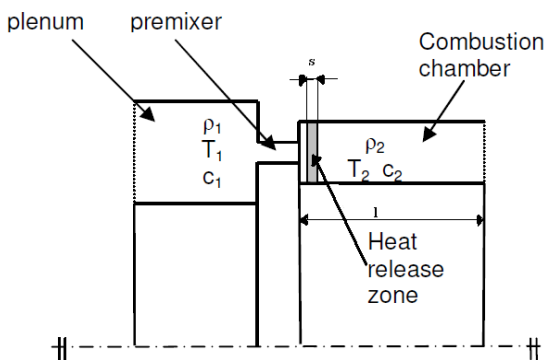

FIG. 13 GEOMETRY OF THE ANNULAR COMBUSTION CHAMBER.

In this case, following the method proposed by Waltz et al. [25], and by Forte et al. [26], only a quarter of the whole annular system, divided by two meridian planes as shown in Fig. 15, is considered as computational domain and discretized in the FEM

analysis in order to reduce the computational effort. The analysis of one quarter of the whole geometry, requires two steps: in the first one a closed end ($u'=0$) is assumed as boundary condition on both the meridian planes (*symmetrical boundary conditions*); in the second step a closed end is assumed on one meridian plane and an open end ($p'=0$) at the other plane (*asymmetrical boundary conditions*). The eigenmodes of the original system are obtained as the ensemble of the eigenmodes obtained in the first and in the second step. Through this approach a smaller domain can be simulated, able to provide the same eigenvalues and eigenmodes of the original geometry. In this test the burners are modeled by means of analytical transfer matrices.

The obtained results for the first four modes are provided in Table 3. Eigenmodes are denoted with the nomenclature *(l,m,n)*, where *l*, *m* and *n* are, respectively, the orders of the pure axial, circumferential and radial modes. The corresponding eigenmodes are shown in Fig. 15.

TABLE 3 VALUES OF THE NORMALIZED FREQUENCIES.

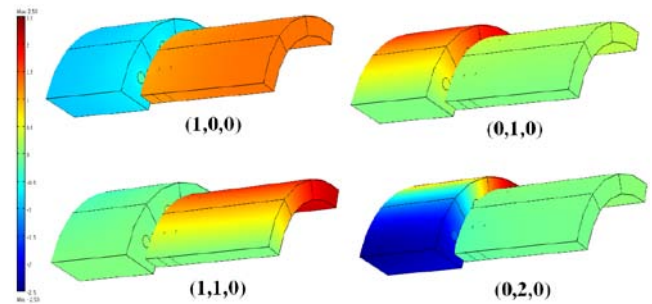| Mode Shape	(1,0,0)	(0,1,0)	(1,1,0)	(0,2,0)
Numerical Results	0.242	0.350	0.576	0.658

FIG. 14 FIRST FOUR ACOUSTIC EIGENMODES OF THE COMBUSTOR.

Fig. 16 clearly shows that the use of the transfer matrix does not alter the acoustic pressure field around the burners.
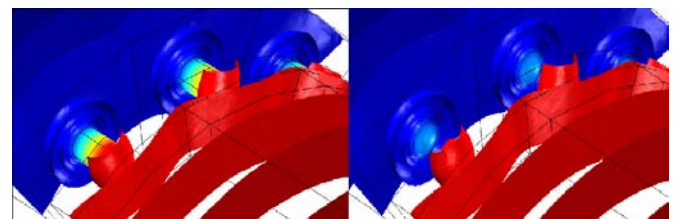

FIG. 15 ACOUSTIC PRESSURE FIELD WITH (LEFT) AND WITHOUT (RIGHT) TRANSFER MATRIX

The influence of the pressure loss coefficient is examined by varying the time delay τ for constant Mach number and β. Two different values of ζ are considered: $\zeta = 0$ and $\zeta = 2$ and the paths of normalized frequency and growth rate are detected for the first circumferential mode in the plenum (Fig. 17) and the first circumferential mode in the combustion chamber (Fig. 18).

Fig. 17 and Fig. 18 show the great importance of the pressure loss coefficient and, generally, of the damping effect which can be applied. An increase of ζ determines a reduction in the growth rate and so an increase in the stability condition. The effect on the frequency has not a great importance, because its variation is negligible.
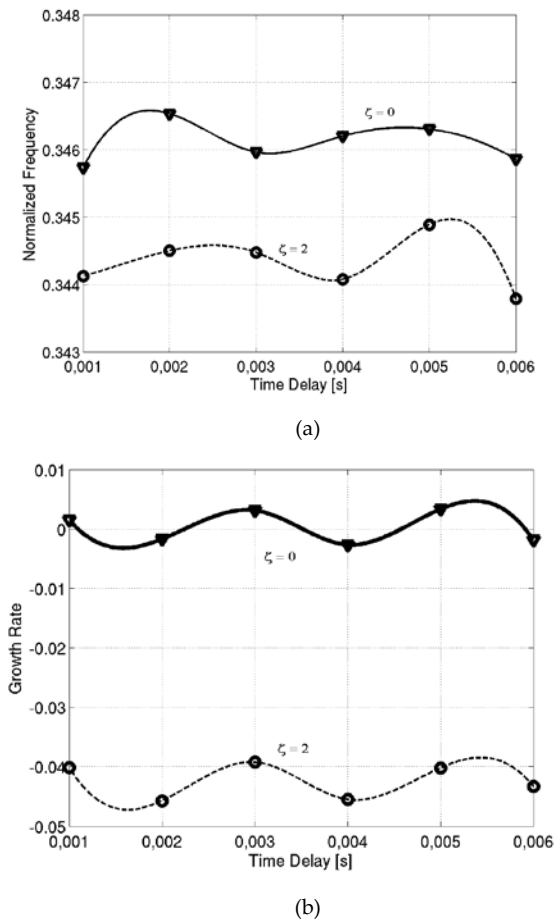

(a)

(b)

FIG. 16 VARIATION WITH THE TIME DELAY OF NORMALIZED FREQUENCY (A) AND GROWTH RATE (B) OF THE FIRST CIRCUMFERENTIAL MODE IN THE PLENUM (FIG. 15)

In the light of these results, it is worth noting the combined effect of time delay and pressure loss coefficient. A combination of these two elements, together with the effect of the Mach number, can be helpful to design the burner with parameters able to lead to stable conditions inside the combustion chamber.

Application to a Practical Annular Combustion Chamber

The idea to model the burners with the transfer matrix method is extended to a practical application. The annular combustion chamber by Ansaldo Energia is examined. An annular plenum is placed at the exit of the axial compressor and is connected to the annular combustion chamber through 24 burners, whose details can be found in [27]. Fig. 19 shows the geometrical configuration and the boundary conditions.
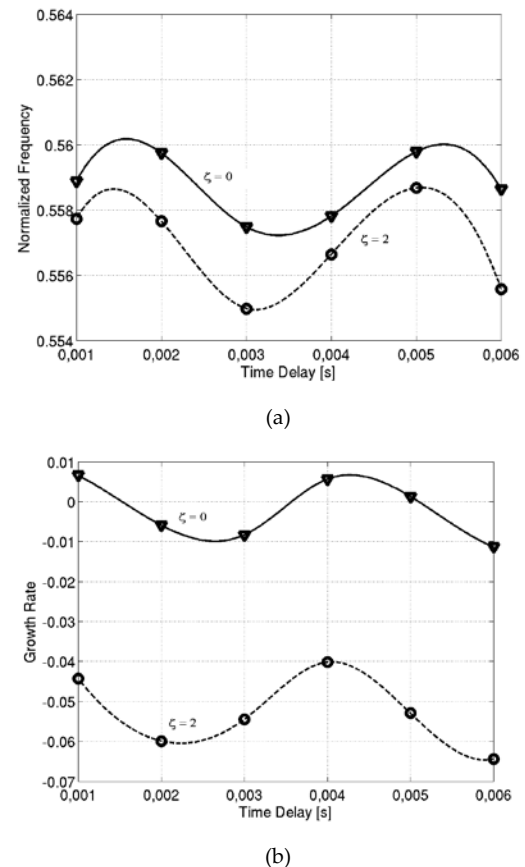

(a)

(b)

FIG. 17: VARIATION WITH THE TIME DELAY OF NORMALIZED FREQUENCY (A) AND GROWTH RATE (B) OF THE FIRST CIRCUMFERENTIAL MODE IN THE COMBUSTION CHAMBER (FIG. 15).
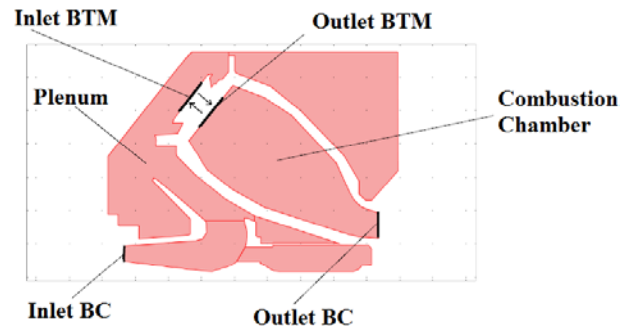

FIG. 18 COMPUTATIONAL DOMAIN AND BOUNDARY CONDITIONS OF THE ANNULAR COMBUSTION CHAMBER BY ANSALDO ENERGIA

In the acoustic analysis, the volume corresponding to the duct of each burner is removed and substituted by a transfer matrix, so that the upstream port of each matrix is the exit from the plenum and the downstream is the inlet of the combustion chamber. Operating conditions are taken from experimental data and from steady flow Reynolds Averaged Navier Stokes (RANS) simulations. A spatial distribution of the temperature is defined using the results of the RANS simulations, which have been imported into the 3D FEM solver. The heat release fluctuations are modeled using the flame model

$$\frac{q'(\mathbf{x})}{\overline{q}(\mathbf{x})} = -n \frac{u_i'(t - \tau(\mathbf{x}))}{\overline{u}_i} \tag{15}$$

In this configuration the reference position i for the acoustic velocity corresponds to the combustion chamber inlet, which is also the burner transfer matrix interface. The mean value of the heat release $\overline{q}$ is expressed as

$$\overline{q} = RR(\mathbf{x}) \cdot LHV \tag{16}$$

where RR represents the spatial distribution of *Rate of Reaction* [$kmol/m^3 s$], *LHV* is the Lower Heating Value [$J/kmol$] of the fuel. The volumetric heat release model defined in Eq.(16) is transferred into the governing equation. In doing so heat release law is applied inside the whole combustion chamber domain, and not in a specific thin domain at the beginning of the combustion chamber as done in the previous models. All the information concerning this modeling in the FEM code are shown in a previous work of ours [28].

An investigation concerning the influence of the pressure gradient Δp inside the burners is carried out. The influence of Δp is considered inside the burner transfer matrix, Eq.(14), and particularly through the pressure loss coefficient ζ. The linear flame response, Eq.(15), with the spatial distribution of the *Rate of Reaction* is used and the complex eigenfrequencies are detected and shown in Fig. 20. The time delay is assumed to be constant, $\tau = 7\ ms$. Three different values of ζ are considered: the pressure loss coefficient corresponding to the design burner ($\zeta = 0.98$), the pressure loss coefficient corresponding to a larger pressure gradient ($\zeta = 2.27$), and a zero pressure loss coefficient.
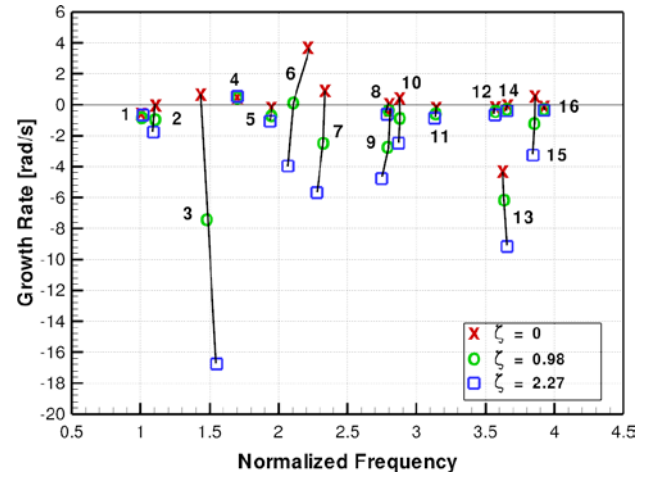

FIG. 19 COMBUSTION CHAMBER MODES FOR DIFFERENT VALUES OF Z WITH THE SPATIALLY DISTRIBUTED FLAME, EQ.(15). *T =7 MS.*

The results in Fig. 20 show damping effects produced by the pressure losses: the increase of ζ determines a general decrease in the growth rate, with some modes moving from the unstable area to the stable one. Not all the modes are subjected to a change in the frequency and in the growth rate, some of them are not influenced. Modes 3, 6 and 7 have the largest variation in the growth rate, moving from an unstable condition without damping ($\zeta = 0$) to a stable one (with $\zeta = 0.98$ and $\zeta = 2.27$). The causes of these results can be found in the fact that Modes 3 and 7 are axial modes that involve propagation of the pressure waves through the burners, whereas Mode 6 is an azimuthal one involving propagation of the pressure waves over the entire combustor system. In general, the modes for which there is an interaction between the waves in the plenum and the waves in the combustion chamber are characterized by a stronger influence from the pressure loss coefficient ζ. On the other hand, ζ has no influence on the modes for which the wave interaction is negligible, such as for Mode 4, and on the modes for which the fluctuations are negligible inside the combustion chamber, such as for Mode 8.

Conclusions

A hybrid technique, based on the combined use of a finite element method and acoustic transfer matrices, is proposed for the simulation of the combustion instability. In order to establish proof of concept, the method is applied to several test cases, from simple cylindrical ones to an annular one and finally to an industrial annular configuration. The substitution of a piece of geometry can be useful when this element has a complex shape, so that it is possible to reduce the computational efforts. The search of the instability

conditions by means of this method yields good results. The modes are caught very well, and the possible differences are due to tridimensional effects which at high frequencies become more and more important despite to the plane wave theory, that can be correctly applied to lower frequencies.

This technique is appropriate to treat complex geometries based on real dimensions and shapes, which are really difficult to be treated by means of analytical methods or acoustic network methods. In this analysis experimental transfer matrix functions and flame transfer functions are successfully applied.

In conclusion, this hybrid technique appears to be a simple tool for the analysis of thermoacoustic combustion instability both in design stage and in check stage, taking the hypothesis of linear acoustic waves. The application of this method in the design stage permits to keep an useful instruments to detect the best operating parameters and the best geometrical shapes and dimensions of the examined system, without the necessity of facing long and expensive experimental tests. In the check stage this method permits to verify the correctness of the assumptions made in the design stage detecting the stable and unstable conditions comparing them to the effective operating conditions.

Acknowledgements

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